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Origin and propagation of cosmic rays in the range 100–1000 GeV

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Abstract. Recent results have indicated that there is a significant anisotropy in the arrival directions of cosmic ray particles having median energy in the range 200–300 GeV. In the present work the assumption is made that the majority of these particles are of Galactic origin and conclusions are drawn about their propagation in the local region of the Galaxy. Observations on cosmic γ rays indicate that there is either a long-term gradient of cosmic ray intensity along the local spiral arm or that the Vela supernova is producing a temporary one; in either case the measured anisotropy indicates that the mean free path for particle scattering is about 7 pc along the local magnetic field direction and probably about half this value in the perpendicular direction.

1. Introduction

For many years it was thought that the anisotropy of cosmic rays having energies in the range 100–1000 GeV was very small ($\lesssim 2\times10^{-4}$) and many authors considered this to be strong evidence in favour of their being of extragalactic origin (see the work of Thambyahpillai 1975, for a summary). Recently, however, Marsden *et al* (1976) have made a re-analysis of the data, including the most recent measurements and a new model of the interplanetary magnetic field, and have derived a much higher anisotropy. Utilizing data from the experiments at Holborn, London and Hobart, Australia, Marsden *et al* argue that the anisotropy $\delta=(I_{max}-I_{min})/(I_{max}+I_{min})$ is $\delta\simeq1\cdot7\times10^{-3}$ and that there is evidence that the maximum intensity is in the direction $l^{II}\simeq270°$ $b^{II}\simeq20°$, i.e. roughly along the spiral arm.

In a previous paper (Dodds *et al* 1975) the present group have made a case for cosmic rays of energy in the range 1–10 GeV being of Galactic origin, arguments being put forward which were based on the distribution of Galactic γ rays. Here we examine the higher energy range, 100–1000 GeV, in the light of the new anisotropy data to see what implications there are for Galactic propagation if these particles, too, are of Galactic origin.

The examination relates to three aspects: a general discussion of earlier studies by Dickinson and Osborne (1974) and Bell *et al* (1974) of the relationship between Galactic properties and the anisotropy, information from studies of the cosmic ray gradient as evinced by γ-ray data and the possibility of a significant flux of cosmic rays from a particular, recent supernova.

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Much of the discussion will relate to motion in the local section of spiral arm in which the solar system is thought to reside. The terminology is that the direction $l^{II} = 90°$ will be termed 'inwards' (inwards in the sense of pointing in the direction of the spiral tightening) and $l^{II} = 270°$ will be termed 'outwards'. In fact, there is uncertainty about the topography of the local spiral arm and, furthermore, the distributions of stars, gas and magnetic fields follow somewhat different patterns. Lequeux (1974) has summarized the situation for the local arm as follows:

gas $l^{II} \simeq 60°–80°$ out to about 1 kpc

stars $l^{II} \simeq 50°$ out to about 1 kpc.

The magnetic field direction, which is of great importance in the present work, has been measured by different techniques and to varying distances. Of relevance to the discussion here is the field direction out to about 400 pc and the data here are given in table 1 for the various techniques.

Table 1. Data on the mean magnetic field direction.

Technique	Field direction	Distance in the Galaxy
Optical polarization	$l^{II} \simeq 80°$ (+ local helical field)	From 300 to 4000 pc
Faraday rotation of pulsars	$l^{II} \simeq 90–110°$	Mean distance 300 pc
Faraday rotation of quasars	$l^{II} \simeq 110°$	Distance $\lesssim$ 2–3 kpc

The conclusion to be drawn is that although the gas and stars (particularly the latter) are distributed along an axis which does not run from 90° to 270° there is quite strong evidence that the magnetic field direction is quite close to this axis.

2. Previous studies

The anisotropy predicted for cosmic rays of Galactic origin depends on models for the production of the particles and for their propagation. Dickinson and Osborne (1974) carried out a detailed analysis of the situation where the sources are supernovae and the propagation can be considered as essentially one-dimensional diffusion. Here, cosmic rays remain in their own magnetic flux tube and propagate along the tube by one-dimensional diffusion due to scattering from minor irregularities in the field, while the flux tubes themselves experience a three-dimensional random walk, with a step size of several tens of parsecs, until they reach a height above the Galactic plane where the gas density is so small that the cosmic rays freely emerge. The model of Dickinson and Osborne almost certainly needs revision because of problems pointed out by Skilling *et al* (1974) related to the likelihood of transfer of a particle from one field line to another on scattering but the model is useful in illustrating the relationships involved.

The authors investigate the time variation of the anisotropy assuming that the observed cosmic rays are produced by supernovae occurring within 30 pc of the axis of the flux tube in which the earth is situated. From this the probability is calculated of our now being at a point in the earth's cosmic ray history when the anisotropy is less than some particular value for various values of the diffusion mean free path along the flux tube. The probabilities of δ being less than 3×10^{-3} (a reasonable value to take in view of the observed value being approximately $1{\cdot}7\times10^{-3}$) are 70%, 23% and 8% for mean free paths $\lambda = 1$, 3 and 10 pc respectively. None of these probabilities is unreasonable. Putting $\delta = 1{\cdot}7\times10^{-3}$ as the median value, the required mean free path is $\lambda = 1$ pc but values as high as about 10 pc are quite clearly also acceptable. As the particles are required to diffuse distances of the order of kiloparsecs along the flux tube before escaping, mean ages of some millions of years are predicted even for these relatively long mean free paths.

In the Dickinson–Osborne model the direction of maximum intensity is essentially that along the local flux tube towards the nearest most recent supernova that contributes particles to the flux tube. Even if there is an average gradient of the frequency of supernovae along the spiral arm, with more on the inward side, this does not necessarily mean that the maximum intensity should be inwards. Thus, there is no objection to the experimental result, with its maximum outwards. The direction of the local flux tube is somewhat arbitrary but, in view of the overall mean field being along the arm, it would be likely to be not far from the axis of the arm.

As an alternative model, mention can be made of that put forward by Bell *et al* (1974). These workers considered the astronomical evidence for irregularities in the magnetic field in the Galaxy and estimated $\lambda \approx 20$ pc, at least for those irregularities of relevance to cosmic rays of energy above about 10^{14} eV. Anisotropic diffusion was considered in which, because of the bulk magnetic field along the spiral arm, the diffusion coefficient along the arm is greater than that perpendicular to it (for energies below 10^{16} eV). The result derived was that the anisotropy should have approximate magnitude 4×10^{-3} and the maximum and minimum intensities should be along the spiral arm. It will be noted that the predicted anisotropy is not much greater than the observed value and the direction of maximum is correct, although the sense of the maximum (inwards or outwards) is not defined. It is appropriate to point out however that, on this model, the particular value for δ arises from an assumption about the position of the earth with respect to the (open) ends of the supposed section of spiral arm. Insofar as this is not known at all the prediction is rather arbitrary. Furthermore, there are problems with the assumption of a constant rate of production in each volume of space. If supernovae (SN) are responsible for the particles, as is usually assumed, and a SN must be within about 30 pc of a field line in order to transfer particles to it (Dickinson and Osborne 1974) then only a few SN will contribute to the cosmic ray flux at the earth at any one time. Thus, the geometrical value for the anisotropy is simply the very long term average and much larger fluctuations in direction and magnitude will result from individual supernovae.

The most important point from the work of Bell *et al*, is its emphasis on anisotropic diffusion. It seems highly likely that the diffusion coefficient perpendicular to the spiral arm (i.e. the direction defined by the magnetic field detection) will be smaller than that along it. The effective mean free path along the arm $\lambda_{\parallel}$ appropriate to the case considered here might be expected to be somewhat less than the 20 pc derived from an examination of astronomical data because at the lower energies under consideration (10^9–10^{12} eV) there will be the effect of smaller and more frequent field irregularities which are unimportant at 10^{15} eV. The idea of preferential streaming along the overall magnetic field direction is crucial to the ideas presented in the next section (cosmic ray gradient model) and is very relevant to our considering the direction $l^{II} = 90°/270°$ as the axis of the spiral arm rather than the star axis.

3. Information from galactic γ rays

3.1. γ-ray data and possible explanations

Measurements on γ rays have shown a preponderance coming from the Galactic plane. The data from the SAS II experiment of Fichtel *et al* (1975) are the most accurate to date and these can be interpreted in terms of contributions from particular sources (specifically Vela and the Crab) and from the results of collisions between cosmic ray protons and nuclei in the interstellar medium (together with a fraction coming from electron bremsstrahlung and inverse Compton interactions). The best documented data relate to γ rays above 100 MeV and the latitude range $-10° < b^{II} < 10°$ (see figure 1) and we confine attention to these quanta. As can be seen, the results show a general enhancement in intensity over the longitude range $40° > l^{II} > 320°$ which has been identified with a particularly high γ-ray emissivity in the region of Galactocentric radius $R \simeq 4$ kpc (see for example the work of Dodds *et al* 1974, Stecker 1975, Strong and Worrall 1976). Interpretation in terms of a greatly increased cosmic ray proton intensity in this region does not follow automatically because of the discovery by Scoville and Solomon (1973) of large concentrations of molecular hydrogen there which could well provide a high target mass so that a large cosmic ray intensity may not be required. Such consensus as there is suggests that there is however some increase in cosmic ray intensity at 4 kpc compared with the local value, and this supports the concept of Galactic origin of cosmic rays at the energies in question ($E \simeq 1$–10 GeV)

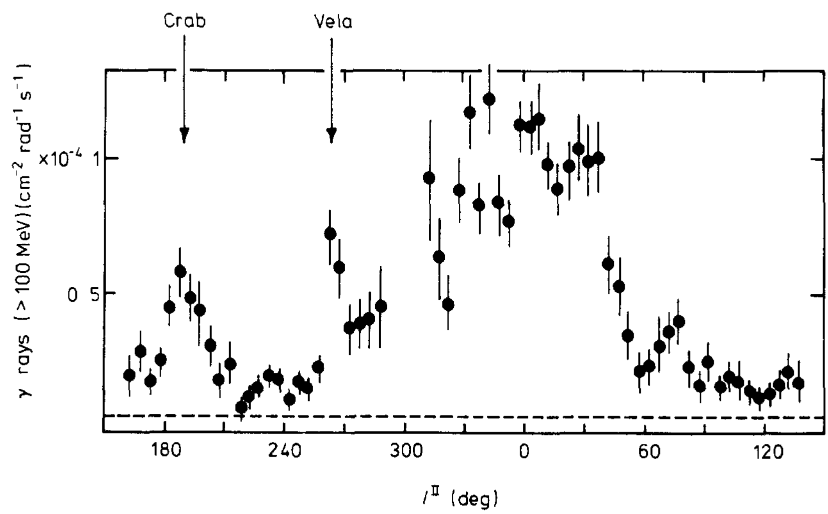

Figure 1. SAS II data of Fichtel *et al* (1975). The data relate to γ rays within the latitude band $-10° < b^{II} < 10°$. The contribution from the diffuse isotropic background is shown as a horizontal broken line. The positions of the Crab and Vela SNR are indicated.

insofar as extragalactic origin would result in a uniform intensity of cosmic rays throughout the Galaxy. Dodds *et al* (1975) have pointed out that better evidence for Galactic origin comes from an analysis of γ rays from the anti-centre direction.

Two facts are immediately apparent from figure 1. The Vela pulsar is generating γ rays at $l^{II} \simeq 270°$ (the majority of the flux in the peak is in fact pulsed: Thompson *et al* 1975) and the region near Vela, say with $240° < l^{II} < 300°$, has a higher average continuum intensity than the equivalent region round $l^{II} = 90°$. It will be shown later than the column density of gas in the direction of $l^{II} \simeq 270°$ is somewhat lower than towards $l^{II} \simeq 90°$ so that two limiting possibilities arise for the continuum, one that these γ rays indicate that the cosmic ray intensity is generally higher in the direction $l^{II} \simeq 270°$, perhaps because of there being habitually a higher rate of supernovae in that direction compared with $l^{II} \simeq 90°$ (or because of statistical fluotuations), and the other that the γ rays arise from pions generated by protons produced in the Vela supernova explosion, the protons only having spread some 100 pc from their source.

In the first situation there will be a long term cosmic ray gradient and corresponding anisotropy: in the second there will be a transitory gradient and anisotropy. In what follows, the two possibilities are considered in turn.

3.2. *Cosmic ray gradient model*

3.2.1. Derivation of the magnitude of the gradient. A study of the geometry of the spiral arm system shows that the bulk of the γ rays produced in proton–interstellar matter (ISM) nucleus interactions in the present case, where Vela is ignored and a smooth cosmic ray density is assumed, come from regions within about 2–3 kpc of the earth. This arises because of the peaking of ISM density in the spiral arms and the (assumed) fall-off in cosmic ray intensity with Galactocentric distance. The relative column densities of ISM in these regions are required before the relative γ-ray intensities (towards 270° and 90°) can be related to relative cosmic ray intensities.

Although we are interested specifically in the directions towards 270° and 90° it is necessary to smooth the data by taking means over significant intervals in view of the statistical uncertainties. Ranges 240°–300° ($\langle 270° \rangle$) and 60°–120° ($\langle 90° \rangle$) appear reasonable. The data of figure 1 indicate that the ratio of the γ-ray fluxes is

$$\frac{I_\gamma(\langle 270° \rangle)}{I_\gamma(\langle 90° \rangle)} \simeq 1{\cdot}6 \pm 0{\cdot}2$$

(the Vela pulsed component has been subtracted).

A number of estimates of the equivalent ratios of thickness of target materials have been made, as follows.

(i) *Neutral hydrogen.* It is likely that neutral hydrogen predominates at the l^{II}, b^{II} values in question and the contribution from this component alone will be considered first. Column densities of neutral hydrogen have been given by Daltabuit and Meyer (1972) from various 21-cm surveys. After normalization of the surveys in common l^{II}, b^{II} regions, the results give $\int n_H \, dl$ ratios averaged over 240°–300° and 60°–90° of 1·0 at $b^{II} = 0°$, 0·82 at $b^{II} = -10°$ and 0·74 at $b^{II} = +10°$.

The mean value of b^{II} required in the analysis is not easy to calculate because of the fall in average cosmic ray intensity with increasing distance from the galactic centre. We require the column density weighted towards the nearby regions—within about 2 kpc—and, in view of the gas thickness being approximately ±150 pc the operative mean value of $|b^{\mathrm{II}}|$ required is about 5°. Thus, the required neutral hydrogen ratio is

$$\frac{\langle \int n_{\mathrm{H}}\, \mathrm{d}l \rangle_{270°}}{\langle \int n_{\mathrm{H}}\, \mathrm{d}l \rangle_{90°}} \simeq 0{\cdot}8.$$

It is useful also to derive ratios from the results of other workers who have used 21-cm data and particular models for the distribution of cosmic rays to predict γ-ray intensities. Paul *et al* (1976) assume a coupling between cosmic rays and gas such that $I_{\mathrm{cr}} \propto \rho_{\mathrm{gas}}$ and their ratio of predicted γ-ray fluxes is 0·70. Bignami *et al* (1975) have made similar assumptions and their ratio is calculated by us to be 0·74. In first approximation the corresponding ratio for the present assumptions, where a coupling between cosmic rays and gas is not assumed, will be about $0{\cdot}72^{1/2} = 0{\cdot}86$.

Figure 2 shows the experimental γ-ray fluxes in the regions in question (smoothed from figure 1) and the predictions by Bignami *et al* (1975) and Paul *et al* (1976) using the assumptions just referred to. The excess of observation over prediction for the range $240° < l^{\mathrm{II}} < 300°$ and the reverse situation for $60° < l^{\mathrm{II}} < 120°$ is clearly marked.

(ii) *Neutral and molecular hydrogen.* The analyses just referred to neglect the contribution of molecular hydrogen which, although not predominant at these l^{II}, b^{II} values, is still significant. Fitzgerald (1968) has given colour excesses of stars at various distances from the earth and these data can be used, following the method of Puget *et al* (1975), to derive total hydrogen column densities. Restricting attention to stars about 2 kpc from the earth, and having $b^{\mathrm{II}} = 0°$ (a legitimate procedure with the distance criterion adopted) the ratio ⟨270°⟩ to ⟨90°⟩ is 0·54.

Data are sparse at $|b^{\mathrm{II}}| > 0°$ but, because the molecular hydrogen is confined to a narrower disc than that the neutral hydrogen (Burton *et al* 1975) the ratio here will tend to that for neutral hydrogen alone. The appropriate ratio is intermediate between the 0·8 or so for neutral hydrogen and the 0·54 just derived but nearer the former because

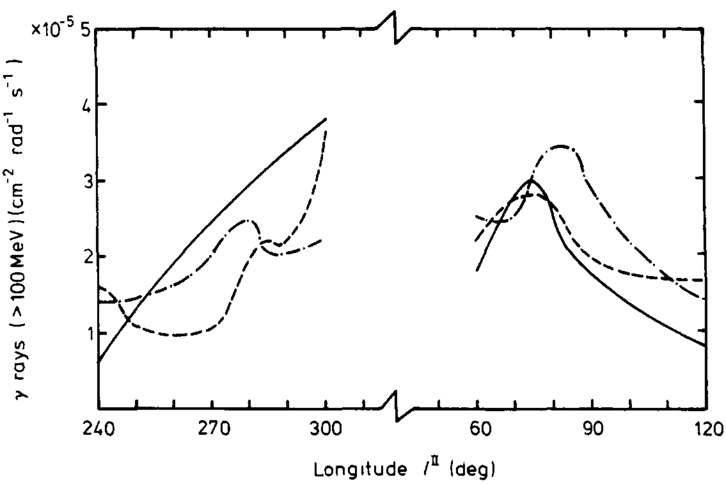

Figure 2. Comparison of observed and predicted γ-ray intensities. The full curve represents the smoothed data of figure 1 (SAS II, Fichtel *et al* 1975) with the Vela peak and the diffuse isotropic background subtracted. The broken curves represent the predictions of Bignami *et al* (1975) and the chain curves the predictions of Paul *et al* (1976). These authors use models in which the cosmic ray intensity is proportional to the ISM density (there are differences in detail between the two treatments).

of the wider (in z) extent of the neutral hydrogen. We adopt a ratio for the amounts of target material of 0·7.

In turn, the ratio of cosmic ray intensities out to approximately 2 kpc in the 270° and 90° directions is $1{\cdot}6/0{\cdot}7 = 2{\cdot}3$.

3.2.2. Relationship between anisotropy and mean free path. The arguments in the previous section indicate that, with the assumptions made, there is a gradient of intensity of cosmic ray protons, of energy in the range 1–10 GeV, along the spiral arm such that the ratio of intensity say 2 kpc outwards (along the arm) to 2 kpc inwards is about 2·3, i.e.

$$\partial I/I\,\partial x \simeq 0{\cdot}2\ \mathrm{kpc}^{-1}.$$

It is not suggested that this gradient continues for any distance round the spiral arm but it does seem to pertain locally at the present time. If this is a long term gradient it will generate an anisotropy in arrival directions of cosmic rays at the solar system. It is immediately obvious that the sense of the anisotropy is the same as that reported by Marsden *et al* for the 200–300 GeV protons and we regard this as interesting.

If we assume that the gradient persists to the higher energies in question, without change of magnitude, then the measured anisotropy can be used to determine $\lambda_{\|}$. The relationship between the quantities is

$$\delta = \lambda_{\|}\,\partial I/I\,\partial x.$$

Thus, $\lambda_{\|} \simeq 8$ pc. It is not surprising that this value is less than the 'astronomical' value of 20 pc (see § 2) which relates to much higher energies.

3.3. Vela model

3.3.1. Energetics of the process. It was remarked in § 3.1 that there is firm evidence for γ rays coming from Vela (see figure 1: the sharp, instrumental resolution limited, spike). These quanta are pulsed and are presumably produced very close to the pulsar itself, probably by electrons since the energy spectrum seems to follow a power law and is not characteristic of generation by way of neutral pions. However, there is circumstantial evidence for protons having been emitted from the object (perhaps in the initial SNR phase) in that some and perhaps all of the excess γ rays with $l^{\mathrm{II}} = 270° \pm 15°$ (compared with $l^{\mathrm{II}} = 90°$) may have been generated by proton interactions in the nearby ISM. In § 3.2 we attributed the excess to a long term excess of cosmic ray intensity towards $l^{\mathrm{II}} = 270°$; here, we take the quite different explanation that the excess is due to Vela alone. This model differs from that of Pinkau (1970) and Higdon and Lingenfelter (1975) who considered proton emission and consequent γ-ray production but confined the protons within the immediate vicinity of the remnant, i.e. they attributed the Vela 'spike' of figure 1 to these particles.

At this stage it is relevant to point out that our suggestion of a significant number ($\approx$1%) of cosmic rays at the earth having come from Vela is not new. Ramaty and Lingenfelter (1971) considered the major local SNR as potential sources of detectable electrons and they concluded that Vela was the most likely. They were able to construct a model in which at least 10% of the electrons at 750 GeV (but with only about 1% at 3 GeV) came from this remnant. One might have hoped to see evidence for electrons some 100 pc or so from Vela by way of their synchrotron radiation and we have studied the 85 MHz data of Yates *et al* (1967) from this point of view. There is a clear peak at the position of Vela itself but no really marked extension out to 15° away from it as might have been expected. There is indeed an extension on the northern side of the Galactic plane but this ridge is usually attributed to the North polar spur. The lack of an obvious halo is not a serious drawback to the idea of cosmic ray electrons (and protons) extending to some 100 pc from the source, however, because of the great sensitivity of the synchrotron flux to the mean field, B, in the emitting region ($I_\nu \propto B^2$). Changes of field from place to place in the Galaxy can well negate or deform the angular variation of synchrotron intensity.

Returning to the problem of protons from Vela, it is necessary firstly to check the energetics of the situation. To maintain the observed total energy density of cosmic rays W_{cr} each supernova must on average produce cosmic rays with total energy $E_{sn} = 2aW_{cr}/\tau\rho_{sn}$, where ρ_{sn} is the surface density of supernovae per unit time in the local part of the Galaxy and τ is the mean storage time of cosmic rays in the confinement region which extends to a distance a on either side of the Galactic plane. The value of τ/a can be inferred from the surmised traversal of 4 g cm^{-2} of interstellar gas by cosmic rays during their storage time in the Galaxy. Provided that a is somewhat greater than 0·15 kpc, the half-thickness at half-density of the gas distribution, $\tau/a \simeq 1{\cdot}6\times10^7/n_0$ yr kpc^{-1}, where n_0 is the gas density in the Galactic plane (in units of nucleon cm^{-3}). In the Vela region we take $n_0 = 1{\cdot}4$ nucleon cm^{-3} made up of equal densities of atomic and molecular hydrogen plus the normal abundances of heavier elements (corresponding to 1 hydrogen nucleus cm^{-3}). If supernovae occur at a rate of 1 per 25 years distributed uniformly in a disc of radius 15 kpc, $\rho_{sn} = 10^{-5}$ kpc^{-2} yr^{-1} and with $W_{cr} \simeq 1$ eV cm^{-3} we obtain $E_{sn} \simeq 9\times10^{61}$ eV. Allowance for a gradient of SNR in the Galaxy, as suggested by Ilovaisky and Lequeux (1972), leads to $\rho_{sn} = 1{\cdot}9\times10^{-5}$ kpc^{-2} yr^{-1} and $E_{sn} \simeq 3\times10^{62}$ eV. The actual emission from Vela can be determined, assuming that the protons are still within 100 pc or so of the object, that is within about ±15° in longitude, as follows. Figure 1 indicates that the excess of flux above 100 MeV is

$$\phi(270°\pm15°) - \phi(90°\pm15°) = (7\pm2)\times10^{-6}\ \text{cm}^{-2}\,\text{s}^{-1}.$$

Making allowance for the higher density of gas which appears to be appropriate towards 90° (see § 3.2)—a rather approximate procedure since it is not clear whether this is appropriate to the actual region near Vela—the amended result is

$$\phi(270°\pm15°) - 0{\cdot}7\times\phi(90°\pm15°) = (8\pm3)\times10^{-6}\ \text{cm}^{-2}\,\text{s}^{-1}.$$

If the shape of the energy spectrum of the protons near Vela at the present time (or rather at the time when the γ rays recorded at earth left that region) is the same as the interstellar spectrum near the earth, I_0, we can use the γ-ray yield factor appropriate to this spectrum given by Dodds *et al* (1976) of $1{\cdot}24\times10^{-25}$ γ rays above 100 MeV per second per hydrogen nucleus (the nucleons in nuclei with $Z>1$, amounting to 40%, are allowed for in this analysis). Working back from the detected flux of γ rays of 8×10^{-6} cm^{-2} s^{-1} at the earth, the emission from the Vela region, at a distance of 400 pc, is $1{\cdot}6\times10^{38}$ s^{-1} (the distance is, in fact, somewhat uncertain). If the average gas density is 1·4 nucleon cm^{-3} the number of target hydrogen nuclei in a sphere of radius 100 pc is 10^{62} and the cosmic ray intensity there will be

$$[(1{\cdot}5\times10^{38})/(1{\cdot}4\times10^{-25}\times10^{62})]I_0,$$

i.e. 10 I_0. The cosmic ray energy density near the earth is 1 eV cm^{-3} so the energy output of Vela is about $10\times10^{62} = 10^{63}$ eV.

We notice that this result is independent of the distance of diffusion perpendicular to the line of sight provided that it is less than the semi-thickness of the gas layer. There is, however, a first-order dependence on this distance through the derivation of the observed flux of γ rays (i.e. the $\pm 15°$ adopted). The near agreement of this estimate of the energy output of Vela with the energy requirement for the Galaxy as a whole is re-assuring; the discrepancy that does exist can be accounted for if Vela is a stronger source of cosmic rays than the average supernova.

3.3.2. Anisotropy. Since the Vela supernova remnant has galactic coordinates $l^{\mathrm{II}} = 263°$, $b^{\mathrm{II}} = -3°$ it is apparent that any anisotropy in the cosmic ray flux due to a contribution from Vela is likely to be close to the direction of that observed by Marsden *et al.* Here we investigate whether one can obtain a diffusion model which explains the magnitude of the anisotropy and is consistent with the known age (from pulsar data), $1{\cdot}1 \times 10^4$ yr, and distance of the remnant (400 pc).

If one assumes cosmic ray propagation by three-dimensional diffusion the density of particles at a position x, y, z at a time t after the release of a total number of particles, S, of a given energy from a source at the origin of the coordinate system is

$$N(x, y, z, t) = \frac{S}{4\pi D_\perp t} \exp\left[-\left(\frac{y^2+z^2}{4D_\perp t}\right)\right] \frac{1}{(4\pi D_\parallel t)^{1/2}} \exp\left[-\left(\frac{x^2}{4D_\parallel t}\right)\right]. \quad (3.1)$$

$D_\parallel$ is the diffusion coefficient in the x direction and $D_\perp$ is the coefficient in the y and z directions. We define the z axis as running perpendicular to the galactic plane and the x axis as running from Vela to the earth along the spiral arm. Observational data on the Galactic magnetic field may be interpreted in terms of a random component of the field superimposed on a regular component of equal magnitude directed along the spiral arm (Osborne *et al* 1973). It is reasonable, therefore, to suppose that $D_\parallel \approx 2D_\perp$. The expression (3.1) will apply provided that in the time t a negligible fraction of particles have diffused to the nearest boundary of the confinement region.

The flux of particles from Vela will be superimposed upon a background flux of particles from older, more distant supernovae. We will consider the background flux to be isotropic. The total density of particles is then

$$N_{\mathrm{tot}} = N + (S\rho_{\mathrm{sn}}\tau/2a) \quad (3.2)$$

where all supernovae contribute S particles.

The anisotropy of cosmic rays at earth ($x = x_{\mathrm{E}}$, $y = 0$, $z = 0$) is given by

$$\delta = \frac{\lambda_\parallel}{N_{\mathrm{tot}}} \frac{\mathrm{d}N_{\mathrm{tot}}}{\mathrm{d}x} = \frac{N}{N_{\mathrm{tot}}} \frac{3x_{\mathrm{E}}}{2ct} \quad (3.3)$$

where $\lambda_\parallel = 3D_\parallel/c$ is the mean free path in the x direction. It follows that

$$\delta = \frac{3x_{\mathrm{E}}}{2ct}\left[\frac{\rho_{\mathrm{sn}}\tau}{2a}(4\pi t)^{3/2}(D_\parallel D_\perp{}^2)^{1/2} \exp\left(\frac{x_{\mathrm{E}}{}^2}{4D_\parallel t}\right) + 1\right]^{-1}. \quad (3.4)$$

The value of τ/a in the expression must be that appropriate to the 300 GeV cosmic rays for which the anisotropy is measured. A reasonable extrapolation of experimental data indicates that cosmic rays of this energy traverse about 2 g cm^{-2} of interstellar gas. Thus $\tau/a = 5{\cdot}7 \times 10^6$ yr kpc^{-1}. Setting $D_\parallel = 2D_\perp$ and with $\rho_{\mathrm{sn}} = 1{\cdot}9 \times 10^{-5}$ kpc^{-2} yr^{-1} we obtain

$$\delta = 0{\cdot}18/[1{\cdot}39 \times 10^9 \, D_\parallel{}^{1{\cdot}5} \exp(3{\cdot}64 \times 10^{-6}/D_\parallel) + 1]$$

where $D_{\parallel}$ is in $kpc^2\,yr^{-1}$. Thus, to account for the observed anisotropy we need $D_{\parallel} = 7{\cdot}7 \times 10^{-7}\,kpc^2\,yr^{-1}$, corresponding to $\lambda_{\parallel} = 7{\cdot}5$ pc. This mean free path is for cosmic rays with energy about 300 GeV, however, a similar mean free path for 3 GeV cosmic rays can account for the observed distribution of γ-ray emission from around Vela. Substituting in equation (3.1) the y and z dependence of the cosmic ray density is $\exp[-(y^2+z^2)/(0{\cdot}13)^2]$, where y and z are in kiloparsecs, and 45% of cosmic rays emitted remain within 0·1 kpc of the x axis (and the energy output of Vela would need to be somewhat greater than the 10^{63} eV referred to earlier). The observed emission, being an integral over the line of sight in the x direction, is, to first order, independent of the distribution in x of the particles.

The longitudinal mean free path, deduced from the anisotropy, is not strongly dependent on the local supernova rate or the assumption that all supernovae produce the same number of particles. For a three times larger local supernova density $\lambda_{\parallel} = 12$ kpc. If the Vela supernova produced, for instance, five times more cosmic rays than an average supernova then $\lambda_{\parallel} = 5$ pc.

4. Conclusions

The observed anisotropy of cosmic rays and the distribution of γ-ray emissivity around the Vela supernova remnant can be accounted for in terms of the production of cosmic rays in the supernova explosion and propagation by three-dimensional diffusion if, in the 3 to 300 GeV energy range, the longitudinal mean free path $\lambda_{\parallel} \simeq 7$ pc, the perpendicular mean free path is about half as big and Vela is a somewhat stronger source of cosmic rays than the average supernova. Alternatively, if the anisotropy is not due to this supernova but is a manifestation of a long-term gradient in the cosmic ray intensity a similar value of $\lambda_{\parallel}$ is indicated provided that the cosmic ray propagation is independent of energy in the 3 to 300 GeV range.

There are three ways to reconcile these relatively long mean free paths with the traversal of a few $g\,cm^{-2}$ of interstellar gas before leaving the Galaxy.

(i) The cosmic ray particles diffuse along flux tubes which random walk some kiloparsecs before reaching a height above the Galactic plane where the particles can escape.

(ii) The region of diffusion extends to a considerable distance from the Galactic plane. For $\lambda_{\perp} \simeq 4$ pc the boundary at which free escape occurs would have to be at a distance from the plane of about 11 kpc.

(iii) Partially reflecting boundaries exist above and below the Galactic plane. The reflection coefficient determines the amount of matter traversed before escape from the trapping region.

The first has been discussed in § 2 where it was pointed out that there are doubts about the validity of the concept of propagation along flux tubes. The second implies the existence of a substantial cosmic ray halo around the Galaxy for which there is little observational support. The energy dependence of the interstellar matter traversal can be accounted for only in an *ad hoc* manner in this picture. In contrast a model of the third type developed by Holmes (1974) does give a matter traversal having an energy dependence close to that observed. Following Skilling (1971) the partially reflecting boundaries separate a 'free' zone near to the Galactic plane, where the gas density is sufficient to damp out cosmic ray generated Alfvén waves, from 'wave' zones above and below the plane where the damping is small and cosmic rays are constrained to stream at close to the Alfvén velocity due to scattering by self-generated waves. The distribution of the gas and the form of the cosmic ray spectrum determine that the separation of the boundaries increases and the reflection coefficient decreases with increasing energy. Within the 'free' zone the divergence of neighbouring field lines, caused by the background spectrum of interstellar turbulence, and the scattering by magnetic mirrors related to interstellar clouds (Skilling *et al* 1974) can give effective three-dimensional diffusion with $\lambda \simeq 7$ pc.

Wentzel (1974) has argued that the additional damping of self-generated waves due to wave–wave interactions gives an upper limit to the energy of individual cosmic rays that can be effectively scattered even in the low density region and that this limit will be in the range 10 to 100 GeV. Skilling (1975) contends, however, that low energy cosmic rays can generate indirectly enough long wavelength Alfvén waves to scatter cosmic rays with energies up to 1000 GeV. Taken at face value the observed anisotropy indicates that the mechanism that produces the partially reflecting boundaries on either side of the Galactic plane is still operating at energies as high as 300 GeV.

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